sults in 'polygon-boden' on the other, every gradation of type of soil-flow may be found, and the combined results of their activities is a transportation of material as important as that of the streams and glaciers.

All the field evidence tends to show that nivation and solifluction, characteristic processes of disintegration and denudation under subarctic or arctic conditions, are of prime importance in the reduction of the high relief of northern Greenland.

[1] Eakin, H. M., *Washington, U. S. Geol. Survey, Bull.* 631, p. 76, 1916.
[2] Mathes, F. E., *Washington, U. S. Geol. Survey, 21st Ann. Rep.*, p. 181, 1899–1900.
[3] Andersson, J. G., *Chicago, J. Geol., Univ. Chicago*, **14,** 1906, p. 91.

ON THE α-HOLOMORPHISMS OF A GROUP

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The term α-holomorphism was introduced by J. W. Young to denote a simple isomorphism of a group G with itself which is characterized by the fact that each operator of G corresponds to its α^{th} power.[1] A necessary and sufficient condition that an abelian group of order g admits an α-holomorphism is that α is prime to g, and J. W. Young proved in the article to which we referred that when a non-abelian group admits an α-holomorphism the $(\alpha-1)^{th}$ power of each of its operators is invariant under the group and the group admits also an $(\alpha - 1)$-isomorphism. Moreover, these conditions are sufficient for the existence of an α-holomorphism.

The object of the present note is to furnish a complete answer to the following question: For what values of α is it possible to construct non-abelian groups which admit separately an α-holomorphism? It will be proved that no such group is possible when α is either 2 or 3, but that for every other positive integral value of α there is an infinite system of non-abelian groups each of which admits an α--holomorphism.

The fact that every group which admits a 2-holomorphism is abelian results directly from a theorem noted in the first paragraph of this article. If a non-abelian group G admits a 3-holomorphism we may represent two of its non-commutative operators by s_1, s_2, and note that as a result of this holomorphism the two dependent equations

$$s_1^3s_2^3 = (s_1s_2)^3, \qquad s_1^2s_2^2 = (s_2s_1)^2$$

must be satisfied. Since s_1^2 and s_2^2 are invariant under G it results directly from the latter equation that $s_1s_2 = s_2s_1$, and hence the assumption that s_1 and s_2 are non-commutative led to a contradiction. That is, *if a group admits a 3-holomorphism it must be abelian.*

We shall now prove that when p is any odd prime number it is possible to construct a non-abelian group whose order is of the form p^m and which admits a $(1 + kp)$-holomorphism, k being an abitrary positive integer. Suppose that k is divisible by $p^{\beta-1}$ but not by p^β. Hence $1 + kp = 1 + hp^\beta$, where h is prime to p. Let t be an operator of order p which transforms an operator s of order $p^{\beta+1}$ into its $(1 + hp^\beta)^{th}$ power. It will be proved that t transforms each operator of the non-abelian group G of order $p^{\beta+2}$ which is generated by s and t into its $(1 + hp^\beta)^{th}$ power.

This proof is an almost direct consequence of the two dependent equations[2]

$$(st)^p = s^p, \quad (st)^{hp^\beta} = s^{hp^\beta}$$

In fact, from these equations it results that t transforms $s^{\alpha_1}t^{\beta_1}$ and s^{α_1} into the same powers. If we form the direct product of the group G just constructed and any abelian group of type (1,1,1, . . .) we clearly obtain another non-abelian group which is such that t transforms each of its operators into a $(1 + kp)$-holomorphism. The group G can therefore be used to construct an infinite system of groups each of which admits such a holomorphism.

To prove the theorem under consideration it is desirable to note that it is possible to construct a non-abelian group whose order is of the form 2^m and which admits a $(1 + 2^\gamma)$-holomorphism whenever $\gamma > 2$. In fact, if s is an operator of order $2^{\gamma+1}$ and if t is an operator of order 2 which transforms s into its $(1 + 2^\gamma)^{th}$ power the non-abelian group of order $2^{\gamma+2}$ which is generated by s and t will clearly satisfy the required condition. Moreover, each one of the infinite system of groups obtained by forming the direct product of the group just constructed and an abelian group of order 2^l and of type (1,1,1,. . .) must likewise satisfy this condition.

It is now easy to establish the general theorem noted in the second paragraph. To construct a non-abelian group which admits an α-holomorphism, $\alpha > 3$, it is only necessary to consider the factors of $\alpha - 1$. When $\alpha - 1$ is of the form 2^m any one of the groups described in the preceding paragraph satisfies the condition in question. When $\alpha - 1$ is not of this form let p be any one of its odd prime divisors and suppose that $\alpha - 1$ is divisible by p^β but not by $p^{\beta+1}$. Hence α is of the form $1 + hp^\beta$ considered above and it has been proved that *whenever $\alpha > 3$ it is possible to construct an infinite system of groups such that each group of this system admits an α-holomorphism.*

[1] J. W. Young, *New York, Trans. Amer. Math. Soc.*, **3**, 1902, (186).

[2] Miller, Blichfeldt and Dickson, *Theory and Applications of Finite Groups*, Wiley, New York, 1916, p. 108.